Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in As Mathematics

8MA0_01 (Public release version)

Resource Set 1: Topic 8

Integration

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Additional Assessment Materials, Summer 2021

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General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1. Find

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x$$

giving your answer in its simplest form.

(4)

1

$$\int\left(\frac{2}{3}x^3 - 6\sqrt{x} + 1\right)\mathrm{d}x = \int\left(\frac{2}{3}x^3 - 6x^{\frac{1}{2}} + 1\right)dx$$

$$= \frac{1}{6}x^4 - 4x^{3/2} + x + C$$

(Total for Question 1 is 4 marks)

2.

Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\int_1^{2\sqrt{2}} f(x)\mathrm{d}x = 16 + 3\sqrt{2}$

(5)

2

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

$$\int f(x) = x^2 + 3x - \frac{12}{x} + C$$

$$\frac{12}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \frac{12\sqrt{2}}{4} \Rightarrow 3\sqrt{2}$$

$$\left[x^2 + 3x - \frac{12}{x}\right]_1^{2\sqrt{2}} = \left[(2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12}{2\sqrt{2}}\right] - \left[1^2 + 3(1) - \frac{12}{1}\right]$$

$$= (8 + 6\sqrt{2} - 3\sqrt{2}) - (-8)$$

$$= \underline{16 + 3\sqrt{2}}$$

(Total for Question 2 is 5 marks)

3. (a) Given that k is a constant, find

$$\int\left(\frac{4}{x^3}+kx\right)dx$$

simplifying your answer.

(3)

③ a) $\int\left(\frac{4}{x^3}+kx\right)dx$

$= -2x^{-2}+\frac{kx^2}{2}+C$

(b) Hence find the value of k such that

$$\int_{0.5}^{2}\left(\frac{4}{x^3}+kx\right)dx=8$$

(3)

b) $\boxed{\int_{0.5}^{2}\left(\frac{4}{x^3}+kx\right)dx=8}$

$= \left[-2x^{-2}+\frac{k}{2}x^2\right]_{0.5}^{2} = \left[-2(2)^{-2}+\frac{k}{2}(4)\right]-\left[-2(0.5)^{-2}+\frac{k}{2}(0.5)^2\right]$

$= \left(-\frac{1}{2}+2k\right)-\left(-8+\frac{k}{8}\right)$

$= \frac{15}{2}+2k-\frac{1}{8}k$

$\Rightarrow \frac{15}{2}+\frac{15}{8}k=8$

$\Rightarrow \frac{15}{8}k=\frac{1}{2}$

$k=\frac{4}{15}$ //

(Total for Question 3 is 6 marks)

4.

Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(4) $\boxed{\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4}$ $\Rightarrow \int \left(\frac{5}{2}x^{-\frac{1}{2}} + 3\right) dx = 4$

a) $\left[5x^{\frac{1}{2}} + 3x\right]_1^k = \left(5\sqrt{k} + 3k\right) - \left(5 + 3\right)$

$\Rightarrow 5\sqrt{k} + 3k - 8 = 4$

$\Rightarrow 5\sqrt{k} + 3k - 12 = 0$

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

(4)

b) $3k + 5\sqrt{k} - 12 = 0$

let $y = k^{\frac{1}{2}}$

$\Rightarrow 3y^2 + 5y - 12 = 0$

$\Rightarrow (3y - 4)(y + 3) = 0$

$\Rightarrow y = \frac{4}{3}$ or $y = -3$

$\Rightarrow \sqrt{k} = \frac{4}{3}$ or $\sqrt{k} = -3$ → no solution as $\sqrt{k}$ cannot be negative.

$\therefore k = \frac{16}{9}$

(Total for Question 4 is 8 marks)

5.

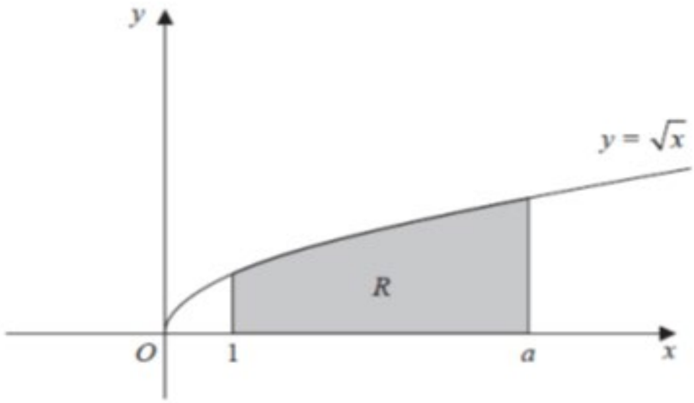


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{x}, \quad x \geqslant 0$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = a$, where a is a constant.

Given that the area of R is 10

(a) find, in simplest form, the value of

(i) $\int_1^a \sqrt{8x}\, dx$

(ii) $\int_0^a \sqrt{x}\, dx$

(4)

5 a) $\int_1^a \sqrt{x}\, dx = 10$

i) $\int_1^a \sqrt{8x}\, dx = \int_1^a \sqrt{8}\sqrt{x}\, dx = \sqrt{8}\int_1^a \sqrt{x}\, dx = 10\sqrt{8} = \underline{\underline{20\sqrt{2}}}$

ii) $\int_0^a \sqrt{x}\, dx = \int_0^1 \sqrt{x}\, dx + \int_1^a \sqrt{x}\, dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1 + 10$

$= \frac{2}{3} + 10$

$= \underline{\underline{\frac{32}{3}}}$

(b) show that $a = 2^k$, where k is a rational constant to be found.

(4)

b) $\int_1^a \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^a = \left[\left(\frac{2}{3}a^{\frac{3}{2}}\right) - \left(\frac{2}{3}\right)\right] = \frac{2}{3}a^{\frac{3}{2}} - \frac{2}{3}$

$\Rightarrow \frac{2}{3}a^{\frac{3}{2}} - \frac{2}{3} = 10 \Rightarrow a^{\frac{3}{2}} = 16$

$\Rightarrow a = 16^{\frac{2}{3}} \Rightarrow a = (2^4)^{\frac{2}{3}} \Rightarrow \underline{\underline{a = 2^{\frac{8}{3}}}}$

(Total for Question 5 is 8 marks)

6.

$$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that g(x) is divisible by (x – 5).

(2)

⑥ a) if $(x-5)$ is a factor of $g(x)$ then $g(5)=0$

$$g(5) = 2(5)^3 + 5^2 - 41(5) - 70$$
$$= 2(125) + 25 - 205 - 70$$
$$= 250 + 25 - 205 - 70$$
$$= 0 \quad \therefore \ (x-5) \text{ is a factor of } g(x)$$

(b) Hence, showing all your working, write g(x) as a product of three linear factors.

(4)

b)

$$\begin{array}{r|l} & 2x^2 + 11x + 14 \\ \hline x-5 & 2x^3 + x^2 - 41x - 70 \\ & -(2x^3 - 10x^2) \\ \hline & 11x^2 - 41x - 70 \\ & -(11x^2 - 55x) \\ \hline & 14x - 70 \\ & 14x - 70 \\ \hline & 0 \end{array}$$

$\longrightarrow$ $g(x) = (2x^2 + 11x + 14)(x-5)$

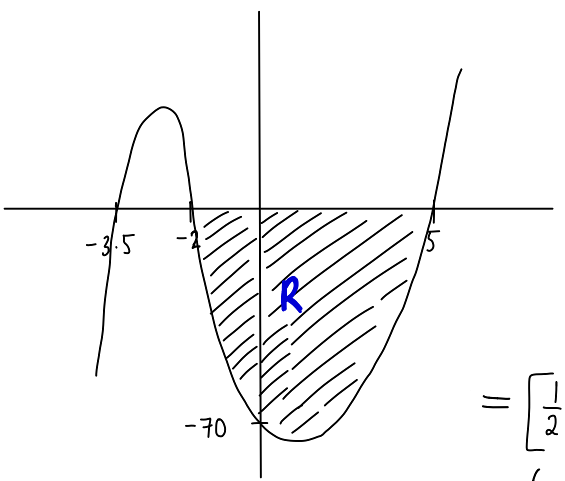
$g(x) = (x+2)(2x+7)(x-5)$ as a product of linear factors

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R.

(4)

c)



$$\int_{-2}^{5} g(x)\,dx = \int_{-2}^{5} (2x^3 + x^2 - 41x - 70)\,dx$$

$$= \left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x\right]_{-2}^{5}$$

$$= \left[\frac{1}{2}(5)^4 + \frac{1}{3}(5)^3 - \frac{41}{2}(5)^2 - 70(5)\right] - \left[\frac{1}{2}(-2)^4 + \frac{1}{3}(-2)^3 - \frac{41}{2}(-2)^2 - 70(-2)\right]$$

$$= \left(-\frac{1525}{3}\right) - \left(\frac{190}{3}\right)$$

$$\Rightarrow R = \left|\frac{-1715}{3}\right| = \frac{1715}{3}$$

(Total for Question 6 is 10 marks)

7.

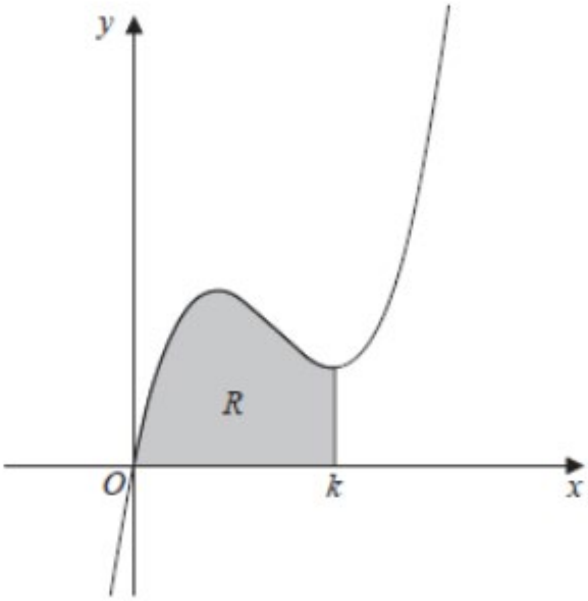


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\frac{256}{3}$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(7)

7 $y = 2x^3 - 17x^2 + 40x$

turning point at $x = k$ (at $\frac{dy}{dx} = 0$) and Area of R is $\frac{256}{3}$.

$\frac{dy}{dx} = 6x^2 - 34x + 40$

$6x^2 - 34x + 40 = 0$

$6k^2 - 34k + 40 = 0$

$3k^2 - 17k + 20 = 0$

$(3k-5)(k-4) = 0$

$k = \frac{5}{3}$ or $k = 4$

By the digram, we are interested in the second turning point so take $\underline{k = 4}$.

$\int_0^4 2x^3 - 17x^2 + 40x \, dx$

$= \left[\frac{x^4}{2} - \frac{17}{3}x^3 + 20x^2 \right]_0^4$

$= \left[\left(\frac{(4)^4}{2} - \frac{17}{3}(4)^3 + 20(4)^2 \right) - (0) \right]$

$= \underline{\underline{\frac{256}{3}}}$ as required.

(Total for Question 7 is 7 marks)

8.

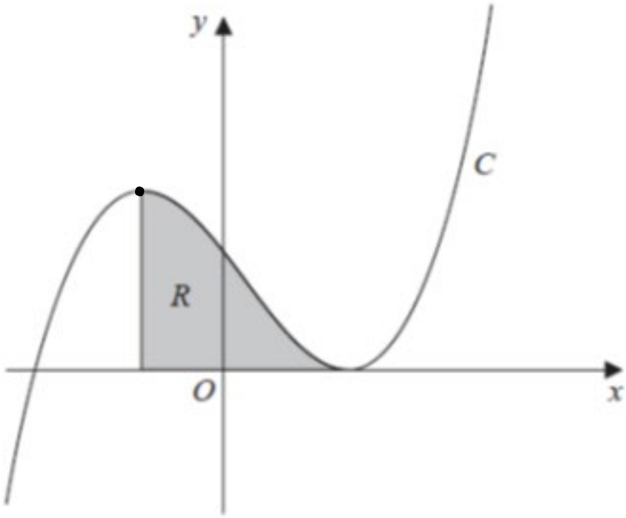


Figure 5

Figure 5 shows a sketch of the curve C with equation $y = (x-2)^2(x+3)$

The region R, shown shaded in Figure 5, is bounded by C, the vertical line passing through the maximum turning point of C and the x-axis.

Find the exact area of R.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(9)

① FIND MAXIMUM TURNING POINT (i.e. when $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$)

$y = (x-2)^2(x+3)$

$y = (x^2-4x+4)(x+3)$

$y = x^3 + 3x^2 - 4x^2 - 12x + 4x + 12$

$y = x^3 - x^2 - 8x + 12$

$\frac{dy}{dx} = 3x^2 - 2x - 8 = 0$

$(x-2)(3x+4) = 0$

$\underline{x = 2}$ or $\underline{x = -\frac{4}{3}}$

at maximum turning point (seen from diagram)

$$\int_{-\frac{4}{3}}^{2} (x^3 - x^2 - 8x + 12)\, dx$$

$$= \left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - 4x^2 + 12x\right]_{-\frac{4}{3}}^{2}$$

$$= \left[\frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 - 4(2)^2 + 12(2)\right] - \left[\frac{1}{4}\left(-\frac{4}{3}\right)^4 - \frac{1}{3}\left(-\frac{4}{3}\right)^3 - 4\left(-\frac{4}{3}\right)^2 + 12\left(-\frac{4}{3}\right)\right]$$

$$= \frac{28}{3} - \left(-\frac{1744}{81}\right) = \frac{2500}{81}$$

Area of R $= \frac{2500}{81}$ //

(Total for Question 8 is 9 marks)

9.

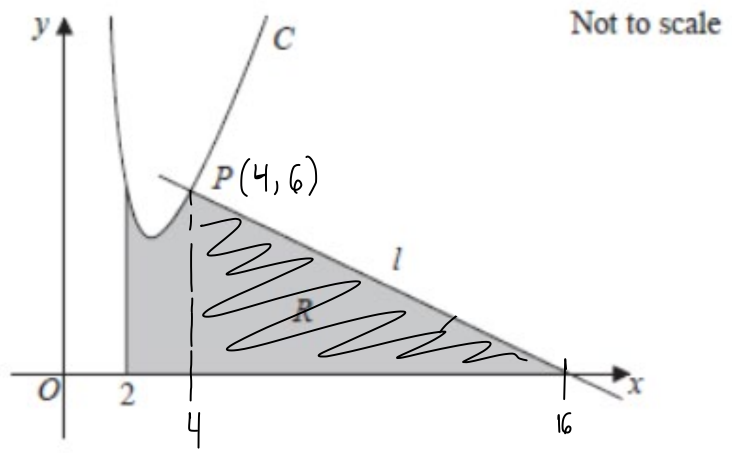


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{32}{x^2} + 3x - 8, \qquad x > 0$$

The point $P\,(4, 6)$ lies on C.
The line l is the normal to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the line l, the curve C, the line with equation $x = 2$ and the x-axis.

Show that the area of R is 46
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

$y = \frac{32}{x^2} + 3x - 8 \Rightarrow y = 32x^{-2} + 3x - 8$

$\frac{dy}{dx} = -64x^{-3} + 3$

gradient at P: $-64(4)^{-3} + 3 = 2$

$\therefore$ gradient of normal $= -\frac{1}{2}$

equation of normal (l) $=$ $y - 6 = -\frac{1}{2}(x-4)$

$y - 6 = -\frac{1}{2}x + 2$

$y = -\frac{1}{2}x + 8$

at $y = 0$, line l: $-\frac{1}{2}x + 8 = 0$

$\frac{1}{2}x = 8$

$\boxed{x = 16}$

Area of triangle $= \frac{1}{2} \times (16-4) \times 6 = 36$

$\boxed{\text{so area of } R = \int_2^4 (32x^{-2} + 3x - 8)\,dx + \text{area of triangle}}$

$\int_2^4 (32x^{-2} + 3x - 8)\,dx = \left[-32x^{-1} + \frac{3}{2}x^2 - 8x\right]_2^4$

$= \left[-32(4)^{-1} + \frac{3}{2}(4)^2 - 8(4)\right] - \left[-32(2)^{-1} + \frac{3}{2}(2)^2 - 8(2)\right]$

$= -16 - (-26) = 10$

$\Rightarrow 10 =$ area under curve (between $x = 2$ and $x = 4$)

$\therefore$ Area of $R = 10 + 36$ (area of triangle)

$= \underline{\underline{46}}$ as required.

(Total for Question 9 is 10 marks)